Yang-Yang thermodynamics of a Bose–Fermi Mixture

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We investigate theoretically the behavior of a one-dimensional interacting Bose-Fermi mixture with equal masses and equal repulsive interactions between atoms at finite temperature in the scheme of thermodynamic Bethe Ansatz. Combining the Yang-Yang thermodynamic formalism with local density approximation in a harmonic trap, we calculate the density distribution of bosons and fermions numerically by treating the radially and axially excited states as discrete and continuous ones, respectively. Our result from exactly solvable solutions may be used as a touchstone of 1D interacting Bose-Fermi mixture for experimental data fitting where mean-field theoretical approaches fail.

I. INTRODUCTION

Recent years have witnessed great development of laser cooling and optical trapping technology, with a remarkable achievement being the quasi one-dimensional (1D) quantum gases realized by tightly confining the particle motion in the other two directions to zero point oscillation [1, 2, 3, 4, 5, 6]. Meanwhile the Feshbach resonance may be used to manipulate the interparticle scattering length by simply tuning an external magnetic field, which enables us to explore the cold atomic gases in the full interaction regime from weakly to strongly interacting limit. In particular, a quasi-1D quantum gas of strongly interacting bosons has been observed in the so-called Tonks-Girardeau (TG) regime [3, 4]. On the other hand, mixture of quantum degenerate gases form novel quantum many-body systems with rich phase structures. Particularly interesting systems are the Bose-Fermi mixtures [7, 22], which rarely occur in nature and have become experimentally accessible with the development of sympathetic cooling [8, 9]. Theoretical investigations on the quasi-1D Bose-Fermi mixture have focused on their phase diagrams and ground-state properties in the scheme of Luttinger liquid theory [10, 11] and Bethe ansatz method [17, 18, 19].

The ground state energy of a 1D system of bosons with repulsive delta-function interaction was first calculated by Lieb and Lininger [12]. Yang and Yang extended this Bethe ansatz method to finite temperature 40 years ago [13]. The Yang-Yang thermodynamic formalism, also known as the thermodynamic Bethe ansatz (TBA), allows to evaluate the thermodynamic properties of the 1D system. It has triggered numerous further investigations and has been generalized later to spin-1/2 fermions [14], mixture of spin-1/2 fermions and bosons [15], and two-component bosons [16], etc. The first direct comparison between experiments and theory based on the Yang-Yang exact solutions was carried out last year [20]

in the weakly interacting Bose gas on an atom chip and for a wide parameter range where conventional models fail to quantitatively describe in situ measured spatial density profiles. In view of the experimental progress, it is thus quite desirable to study theoretically the thermodynamic properties of the quasi-1D Bose-Fermi mixture, which might provide theoretical guidance on the potential experimental implementation. From pure theoretical point of view, it is also very interesting to study how robust of the Bose-Fermi phase separation, which was predicted to appear in the limit of zero temperature [17], against the temperature effect.

In this paper, we combine the TBA and local density approximation (LDA) to study the density distribution of a mixture of bosons and polarized fermions trapped in a harmonic trap at finite temperature. Here we have the temperature, particle number of bosons and fermions, and the interaction strength as variables. We find that the observation of Bose-Fermi phase separation requires even lower temperature attainable by present cooling techniques.

The article is organized as follows. In Section II, we rederive the thermodynamic Bethe ansatz solution by means of Yang-Yang thermodynamic formalism for Bose-Fermi mixture where both bosons and fermions are spin polarized. Section III describes our numerical procedure on how to numerically evaluate the density distribution of bosons and fermions in a harmonic trap at finite temperature. We treat in a different way for radially and axially excited states. Finally in Section IV, we analyze the low temperature behavior of the 1D Bose-Fermi mixture for realistic experimental situation and make concluding remarks in Section V.

II. THERMODYNAMIC BETHE ANSATZ OF BOSE-FERMI MIXTURE

We study a 1D interacting Bose-Fermi mixture on a line of length L with periodic boundary conditions, de-

scribed by the Hamiltonian

$$H = \int_0^L dx \left\{ \frac{\hbar^2}{2m_b} \partial_x \Psi_b^\dagger \partial_x \Psi_b + \frac{\hbar^2}{2m_f} \partial_x \Psi_f^\dagger \partial_x \Psi_f \right. $$
$$\left. + \frac{1}{2} g_{bb} \Psi_b^\dagger \Psi_b^\dagger \Psi_b \Psi_b + g_{bf} \Psi_b^\dagger \Psi_f^\dagger \Psi_f \Psi_b \right\}, \qquad (1)$$

where Ψ_b, Ψ_f are field operators for a boson of mass m_b and for a fermion of mass m_f, and g_{bb}, g_{bf} are boson-boson and boson-fermion interaction strengths, respectively. Fermions are spin-polarized so that Pauli principle excludes their s-wave interaction ($g_{ff} = 0$). This model is exactly solvable for equal masses and equal repulsive boson–boson and boson–fermion interaction strengths, i.e.

$$m_b = m_f = m, \qquad g_{bb} = g_{bf} = g. \qquad (2)$$

Although an exact solution is available only under conditions (2), deviations slightly from this integrable line are expected not to dramatically change the characteristic properties of the system, e.g., the phase separation. Following Lai-Yang's original convention, we assume $2m = 1$ and $\hbar = 1$ and write the Hamiltonian (1) in its first quantization form

$$H = -\sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + 2c \sum_{i<j} \delta(x_i - x_j), \qquad (3)$$

with $c = mg/\hbar^2$. Among the N particles there are M bosons and the rest of them are fermions. The many-body wave function is supposed to be symmetric under odd permutations with respect to indices $i = \{1, ..., M\}$ (bosons) and antisymmetric with respect to $i = \{M + 1, ..., N\}$ (fermions).

For periodic boundary conditions, Imambekov and Demler obtained the following set of Bethe Ansatz equations (BAE) [17]

$$\exp(ik_j L) = \prod_{\beta=1}^M \frac{k_j - \Lambda_\beta + ic/2}{k_j - \Lambda_\beta - ic/2}, \qquad j = 1, ..., N, \quad (4)$$

$$1 = \prod_{i=1}^N \frac{k_i - \Lambda_\alpha + ic/2}{k_i - \Lambda_\alpha - ic/2}, \qquad \alpha = 1, ..., M, \qquad (5)$$

where the momenta $k_1, ... k_N$ are a set of unequal numbers, and spectral parameters $\Lambda_1, ... \Lambda_M$ are the analogs of the momenta. It has also been proved that all solutions of (4-5) are always real, which simplifies greatly the analysis at finite temperature.

For thermodynamics at finite temperature we use the BAE to derive a set of nonlinear integral equations, i.e. TBA equations, which describes the thermodynamics of the model at finite temperature. Taking logarithm of the BAE (4-5), we arrive at the following discrete Bethe

ansatz equations

$$k_j L = 2\pi I_j + \sum_{\beta=1}^M \theta(2k_j - 2\Lambda_\beta),$$

$$2\pi J_\alpha = \sum_{i=1}^N \theta(2k_i - 2\Lambda_\alpha), \qquad (6)$$

with $\theta(k) = -2\arctan(k/c)$. Here I_j and J_α are integer or half integer quantum numbers (depending on the parity of M and N), which play the role of quantum numbers for the momentum k and spectral parameter Λ respectively. For a particular configuration, if an arbitrary quantum number is chosen, it is either occupied (in the set of quantum numbers for the system), called a root, or not occupied, called a hole. In the thermodynamic limit, the distributions of momentum and spectral parameter become dense, and it is convenient to introduce the density functions of roots and holes, respectively. We denote with $\rho(k)$ and $\rho_h(k)$ the density functions of the momentum k and holes, and with $\sigma(\Lambda)$ and $\sigma_h(\Lambda)$ the density functions of spectral parameter Λ and its holes. They are defined by

$$L(\rho(k) + \rho_h(k)) dk = dI,$$
$$L(\sigma(\Lambda) + \sigma_h(\Lambda)) d\Lambda = dJ.$$

Differentiate equation (6) with respect to k and Λ separately, we obtain a set of coupled integral equations

$$\rho(k) + \rho_h(k) = \frac{1}{2\pi} + \frac{1}{2\pi} \int_{-\infty}^\infty K(k, \Lambda) \sigma(\Lambda) d\Lambda,$$

$$\sigma(\Lambda) + \sigma_h(\Lambda) = \frac{1}{2\pi} \int_{-\infty}^\infty K(\Lambda, k) \rho(k) dk, \qquad (7)$$

where

$$K(x, y) = \frac{4c}{c^2 + 4(x - y)^2}.$$

The total number of particles and that of bosons per unit length can be obtained by integrating the density functions of momentum and spectral parameter as follows

$$N/L = \int_{-\infty}^\infty \rho(k) dk, \qquad M/L = \int_{-\infty}^\infty \sigma(\Lambda) d\Lambda, \quad (8)$$

while the energy of the system per unit length is given by

$$E/L = \int_{-\infty}^\infty k^2 \rho(k) dk.$$

With the help of the approach first introduced by Yang and Yang, the entropy of the present model at finite temperature is

$$S/L = \int [(\rho + \rho_h) \ln(\rho + \rho_h) - \rho \ln \rho - \rho_h \ln \rho_h] dk$$

$$+ \int [(\sigma + \sigma_h) \ln(\sigma + \sigma_h) - \sigma \ln \sigma - \sigma_h \ln \sigma_h] d\Lambda.$$

The Gibbs free energy of the model is then defined by $F = E - TS - \mu_F (N - M) - \mu_B M$, where μ_F and μ_B are two Lagrange multipliers, and T is the temperature. In order to arrive the thermal equilibrium, we minimize the free energy with respect to the density functions $\rho(k)$ and $\sigma(\Lambda)$ subject to the constraint (7). In addition, the numbers of fermions and bosons are kept to be constants respectively. It can be proved rigorously that μ_F and μ_B are the chemical potentials of fermions and bosons, respectively.

Applying the minimum condition $\delta F = 0$ gives rise to the following nonlinear integral equations, i.e. TBA equations

$$\epsilon(k) = -\mu_F + k^2 - \frac{T}{2\pi}$$
$$\times \int_{-\infty}^{\infty} K(k, \Lambda) \ln\left(1 + \exp\left(-\varphi(\Lambda)/T\right)\right) d\Lambda,$$

$$\varphi(\Lambda) = \mu_F - \mu_B - \frac{T}{2\pi}$$
$$\times \int_{-\infty}^{\infty} K(\Lambda, k) \ln\left(1 + \exp\left(-\epsilon(k)/T\right)\right) dk, \quad (9)$$

where we have defined

$$\begin{aligned} \exp(\epsilon(k)/T) &= \rho_h(k)/\rho(k), \\ \exp(\varphi(\Lambda)/T) &= \sigma_h(\Lambda)/\sigma(\Lambda), \end{aligned} \quad (10)$$

and the set of equations (7) becomes

$$2\pi\rho(k)(1 + \exp(\epsilon(k)/T)) = 1 + \int_{-\infty}^{\infty} K(k, \Lambda)\sigma(\Lambda) d\Lambda,$$

$$2\pi\sigma(\Lambda)(1 + \exp(\varphi(\Lambda)/T)) = \int_{-\infty}^{\infty} K(\Lambda, k)\rho(k) dk. \quad (11)$$

The density functions $\rho(k)$ and $\sigma(\Lambda)$ can be obtained by solving the above coupled integral equations (9) and (11). The TBA approach described here is universal for discussing the thermodynamics of 1D integrable model. Once T, c, μ_F and μ_B are determined, all thermodynamic properties are known. For instance, the pressure and the free energy are

$$P = \frac{T}{2\pi} \int_{-\infty}^{\infty} \ln\left(1 + \exp\left(-\frac{\epsilon(k)}{T}\right)\right) dk,$$
$$F = -PL + M\mu_B + (N - M)\mu_F. \quad (12)$$

III. LOCAL DENSITY APPROXIMATION AND NUMERICAL PROCEDURE

Starting from the solution to the TBA equations derived above for a uniform 1D Bose-Fermi mixture in thermal equilibrium, in this section we aim to find the numerical results for the finite-temperature density distributions of bosons $n_B(x)$ and fermions $n_F(x)$ under the local density approximation (LDA).

Firstly we describe our iteration process in solving the TBA integral equations. Due to the fact that the TBA equations are a set of coupled nonlinear integral equations there exist no closed analytical solutions for them. Nonetheless, numerically this is in principle a well controllable problem (it becomes, however, quite complex for an increasing number of particle types) and we here solve the equations by iteration. The convergence of this procedure to a solution and its very existence have been investigated most naturally by means of the Banach fixed point theorem [21].

The iteration process is as follows. For given T, c, μ_F and μ_B, we initialize $\epsilon(k)$ and $\varphi(\Lambda)$ on the rhs of equations (9) with the corresponding zero-temperature trial functions $\epsilon^{(0)}(k) = -\mu_F + k^2$ and $\varphi^{(0)}(\Lambda) = \mu_F - \mu_B$, respectively. In a first step, we obtain $\epsilon^{(1)}(k)$ and $\varphi^{(1)}(\Lambda)$ on the lhs of equations (9) and let them be the new trial functions. The scheme continues with updates to $\epsilon^{(n)}(k)$ and $\varphi^{(n)}(\Lambda)$ with $n = 1, 2, ...$. When the relative error between $\epsilon^{(n)}(k)$ and $\epsilon^{(n+1)}(k)$ and that between $\varphi^{(n)}(\Lambda)$ and $\varphi^{(n+1)}(\Lambda)$ reach a small quantity, e.g., 10^{-20}, sufficient convergence is obtained and $\epsilon^{(n)}(k)$ and $\varphi^{(n)}(\Lambda)$ are considered as the solutions of TBA equations. We then put these solutions into equations (11), and meanwhile the initial trial density functions $\rho(k)$ and $\sigma(\Lambda)$ is set as $\rho^{(0)}(k) = 1/2\pi(1 + \exp(\epsilon(k)/T))$ and $\sigma^{(0)}(\Lambda) = 0$, respectively. With the same iteration process, we can obtain the solution of equations (11), the integration of which gives the particle density. Note that, to insure the accurateness of the integration in the equations (9) and (11) for very small interaction strength c, the integrand should be divided into more parts with interpolation method firstly and then integrated numerically.

Experimentally 1D gases are usually achieved by trapping the atoms in a tight harmonic trap with strong transverse confinement and weak confinement along the axis, $\omega_\perp \gg \omega_{//}$. The main parameters from the experiment [20] on Yang-Yang thermodynamics of ^{87}Rb atoms in the $|F = 2, m_F = 2\rangle$ hyperfine state are adopted here. The Bose-Fermi mixture is trapped in a harmonic potential with $\omega_\perp/2\pi = 3280 Hz$, $\omega_{//}/2\pi = 8.5 Hz$.

The numerical procedure is as follows: The total linear density relies on two contributions, i.e. that from the radial ground state and that from the radially excited states. The density of atoms populated in the radial ground state is obtained by solving the TBA equation, while the atoms in the radially excited states are treated *discretely* as an independent ideal 1D Bose(Fermi) gas in thermal equilibrium. This is because our temperature here is on the order of the radial level splitting, $\hbar\omega_\perp/k_B = 157.4 nK$, so that the fraction of the atoms in radially excited states should not be neglected. Along the axis, the energy level splitting, $\hbar\omega_{//}/k_B = 0.4 nK$ is so small that we can use the LDA to account for the axial potential via a *continuously* varying chemical potential $\mu(x) = \mu - V(x)$. In this way, we obtain the total linear densities of bosons $n_B(x)$ and fermions $n_F(x)$ in

the magnetic trap which may be used to fit the experimental data from absorption images.

For the radial ground state, the TBA equations for uniform gas can be applied locally to the trapped gas if the condition for the LDA are met. One assumes that in slowly varying external harmonic trap the chemical potentials of bosons and fermions are changed into

$$\mu_B(x) = \mu_B - \frac{1}{2}m\omega_{//}^2 x^2,$$
$$\mu_F(x) = \mu_F - \frac{1}{2}m\omega_{//}^2 x^2. \tag{13}$$

Here, μ_B and μ_F are chemical potentials for bosons and fermions in the center of the harmonic trap. Replacing the chemical potentials in eqs. (9) by their LDA values, we can obtain numerically the density functions $\rho(\mu_B, \mu_F, x, k)$ and $\sigma(\mu_B, \mu_F, x, \Lambda)$ by iteratively solving the TBA equations (9) together with the constraint (11). The integration of these density functions yields the axial density distributions

$$n_B^{TBA}(x) = \int_{-\infty}^{\infty} \sigma(\mu_B, \mu_F, x, \Lambda)\, d\Lambda \tag{14}$$

for bosons and

$$n_F^{TBA}(x) = \int_{-\infty}^{\infty} \rho(\mu_B, \mu_F, x, k)\, dk - n_B^{TBA}(x) \tag{15}$$

for fermions, which are the LDA revisions to their uniform counterparts, i.e. eqs. (8).

Similar strategies are applied to acquire the densities of bosons and fermions at the radially excited states. For bosons it can be expected that the interaction will significantly affect only the distribution in the radial ground state, while the population in the radially excited states can be well described by the distribution of ideal Bose gas. Fermions in the ground state do not interact with each other due to the Pauli exclusive principle. Thus for fermions, the chemical potential μ_F can be even larger than $\hbar\omega_\perp$, with the population in the radially excited states described by the distribution of ideal Fermi gas. The radially excited states are $(j+1)$-fold degenerate, i.e. for each radial quantum number $j \geq 1$ there are $j+1$ excited states sharing the same energy. We treat each excited state as an independent ideal 1D Bose or Fermi gas in thermal equilibrium with the chemical potential of the gas in the radial ground state given by

$$\mu_B^j(x) = \mu_B(x) - j\hbar\omega_\perp,$$
$$\mu_F^j(x) = \mu_F(x) - j\hbar\omega_\perp, \tag{16}$$

respectively. The density distributions in radially excited state (j) are

$$n_B^j(x) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \frac{1}{\exp\left[\left(\frac{\hbar^2}{2m}k^2 - \mu_B^j(x)\right)/k_B T\right] - 1}\, dk,$$
$$n_F^j(x) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \frac{1}{\exp\left[\left(\frac{\hbar^2}{2m}k^2 - \mu_F^j(x)\right)/k_B T\right] + 1}\, dk. \tag{17}$$
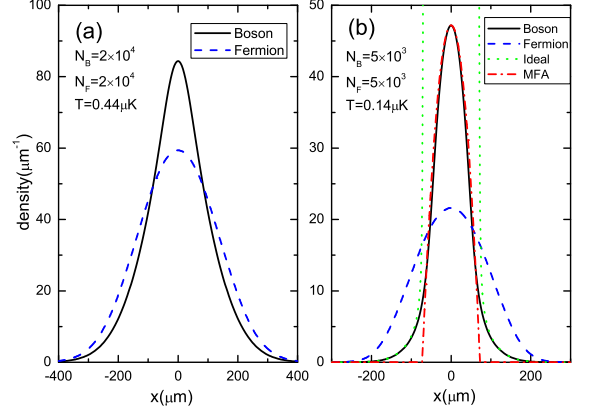

FIG. 1: (Color online) Bose and Fermi density distributions at high (a) and low (b) temperatures for weakly interacting mixture. The interaction strength is chosen as $c_0 = 0.30\mu m$. Yang-Yang thermodynamics is clearly seen for lower temperature as a narrow density peak. Also shown are the ideal Bose-gas profile and that from a mean field theory in the Thomas-Fermi approximation with the same peak density.

The total linear densities are given by summing over the TBA results for radial ground state (14, 15) and the ideal-gas results for radially excited states (17)

$$n_B(x) = n_B^{TBA}(x) + \sum_{j=1}^{\infty} (j+1) n_B^j(x),$$
$$n_F(x) = n_F^{TBA}(x) + \sum_{j=1}^{\infty} (j+1) n_F^j(x). \tag{18}$$

IV. LOW TEMPERATURE BEHAVIOR AND EXPERIMENTAL CONSIDERATIONS

Here, as an example which is experimentally accessible, we show the linear density of atomic clouds with 3D scattering length a equal to $100a_B$ (a_B is Bohr radius) and the mass of the atoms, both boson and fermion, being chosen as that of ^{87}Rb. 1D Bose–Fermi mixture have not attracted much attention until recently, when it became possible to realize such systems in experiments with cold atoms [22]. Experimentalists care more about the heteronuclear Bose-Fermi mixture, such as ^{87}Rb-^{40}K, in which case the heteronuclear interactions can be tailored by means of Feshbach resonances. Bose-Fermi mixture system, however, may be composed of isotopes of atoms whose mass difference is very small, such as ^{6}Li–^{7}Li, or ^{86}Rb-^{87}Rb, etc. The exactly solvable case considered here is relevant to current experiments, and can be used as a benchmark to check the validity of different approaches.

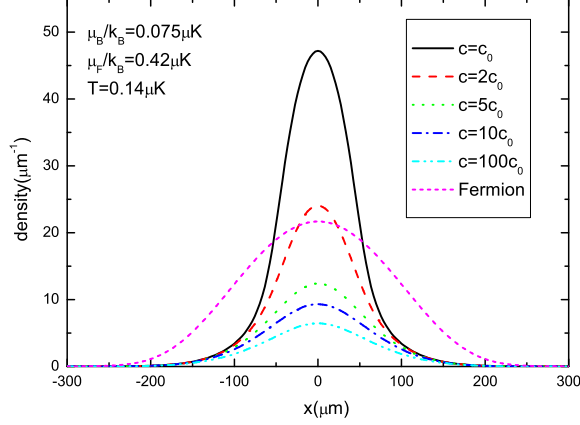We first consider the weakly interacting mixture with

FIG. 2: (Color online) Bose and Fermi density distributions with fixed chemical potential in the trap center and for different interaction strengths. When the interaction strength increases, less number of bosons are needed in the gas to achieve the chemical potential, while the number of fermions in the mixture almost remain unchanged.

FIG. 3: (Color online) Bose and Fermi density distributions at different temperatures for large interaction strength $c = 100 \mu m^{-1}$. Phase separation appears at very low temperature.

the effective 1D coupling strength expressed through the 3D scattering length a as $c = 2m\omega_\perp a/\hbar$ if $a \ll (\hbar/m\omega_\perp)^{1/2}$ [5]. We denote as c_0 the coupling strength for bare ^{87}Rb background scattering which is approximately $0.30 \mu m^{-1}$. In Figs. 1(a) and (b), we show the linear density of atomic clouds for N_B bosons (black solid line) and N_F fermions (blue dashed line) in the magnetic trap for different temperatures. Experimentally these data may be obtained by absorption imaging and integrating the atom number along z-axis. At high temperature ($T = 0.44\mu K$), population in excited states contribute a lot to the density and the result from Yang-Yang formalism for ground state is only a small fraction, hence is not visible. The Yang-Yang thermodynamics is clearly seen for lower temperature ($T = 0.14\mu K$) as a narrow density peak for bosons, where both the ideal-gas and the mean field distributions in the Thomas-Fermi approximation fail to quantitatively describe the spatial density profiles. We also find that in the weakly interacting limit the number of fermions does not affect the density profile of the bosons very much. For instance, one can hardly discern the difference in the distributions of 5×10^3 bosons when we include in the mixture 5×10^2, 5×10^3 or 5×10^4 fermions. This can be attributed to the relatively small number of fermions in the ground state.

The interaction in the mixture, on the other hand, may be enhanced greatly by the so-called confinement induced resonances [5]. Additionally, all interactions can be tuned using available Bose-Fermi Feshbach resonances [22]. In Fig. 2 we illustrate how the bosonic density distribution will change with the increase of interaction strength. For fixed chemical potentials of Bose and Fermi gas, as in-
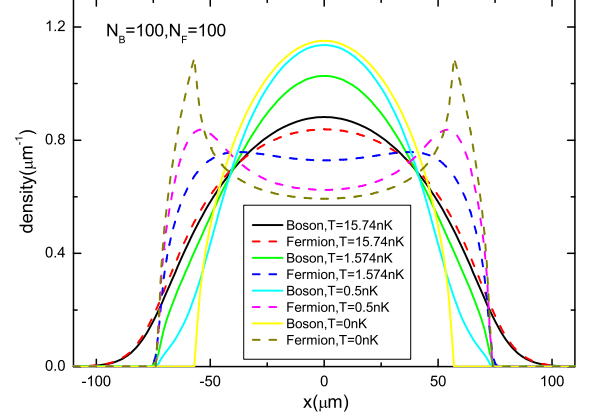
teractions get stronger, the density of Bose gas decreases very quickly, while the density of fermi gas keeps unchanged. This can be explained as follows. In determining the density of the atomic gas, Pauli exclusive principle plays a more important role than the Bose-Fermi interaction g_{bf} for atoms in the ground state (even in the strongly interacting limit) as a result of most fermions occupying the excited states.

Imambekov and Demler [17] predicted the existence of the Bose-Fermi phase separation at very low temperature and very strongly interacting limit, i.e. the relative distribution of bosons and fermions changes with interaction and the Fermi density shows strong nonmonotonous behavior for strong interactions. They firstly get the magnon energy spectrum for large γ ($\gamma = c/n$ and n is density) and then use the local density approximation and energy spectrum to obtain the density distribution. There is, however, no obvious signature of this phase separation in our Fig. 2 which is plotted for $T = 0.14\mu K$ even for very large interaction strength. Clearly the temperature smears off this many-body quantum effect. To observe the phase separation, we need further cool the atomic gas to even lower temperature. In Fig. 3, we compare the density profiles for a mixture of 100 bosons and 100 fermions in the strongly interacting limit $c = 100\mu m^{-1}$ for different temperatures, some of which are beyond the current experimental reach. One clearly sees that the phase separation appears at nK temperatures. Also shown are the zero temperature result which is obtained analytically from Bethe ansatz method [17].

Finally, let us see how the number of fermions would affect the Bose density distribution. In the weakly interacting limit, we do not observe significant change in the density profile of the bosons when we change the number of fermions. However, when both the strongly interacting
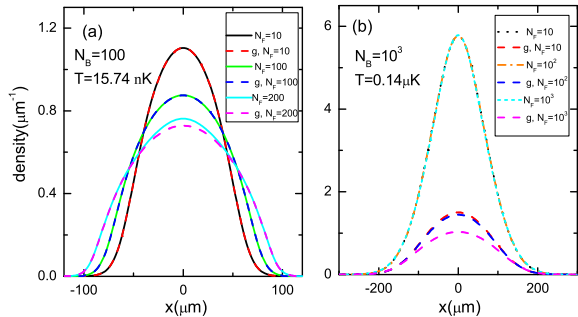
FIG. 4: (Color online) Modification of bosonic density by different number of fermions in the mixture for large interaction strength $c = 100\mu m^{-1}$. In ultra-low temperature (nK) few bosons would populate on the radially excited states. For more fermions in the mixture, bosons in the trap center move to the edge. At slightly higher temperature, the total density remains unchanged. Here "g" stands for density profile of atoms in the ground state.

and ultracold conditions are met, the presence of more fermions drastically alter the density of bosons. We show in Fig. 4(a) the modification of bosonic density by different number of fermions. More fermions produce stronger Fermi pressure, which tends to flatten the Bose density when the interactions between bosons and fermions are strong. We also noticed that the ground state density (dashed lines) coincides almost perfectly with the total density (solid lines) for $N_F = 10-100$, indicating that all bosons $N_B = 100$ occupy the radial ground state. One can see the distinction appears for even larger number of fermions, e.g., for $N_F = 200$, the fattest two curves do not match each other any more. Temperature again blurs the quantum effect from increasing the number of fermions,

i.e. the total density remains unchanged although that of atoms in the ground state indeed decrease for more involved fermions as can be seen in Fig. 4(b).

V. CONCLUSION

In summary, we have theoretically studied the quasi-1D system of Bose-Fermi mixture with equal repulsion between atoms at finite temperature. For the mixture system in a harmonic trap, we calculate the density distributions of bosons and fermions numerically by using the combination of LDA and TBA and treating the radially and axially excited states as discrete and continuous ones, respectively. As the typical temperature for the ultracold gas is on the order of the radial level splitting, the density of atoms populated in the radial ground state is obtained by solving the TBA equation, while those in the radially excited states are treated discretely as an independent ideal 1D Bose gas in thermal equilibrium. The density distributions of bosons and fermions are thus calculated in various experimental situations and the effects due to the interaction strength, particle numbers and temperature are discussed. Our results show that the phase separation between bosons and fermions takes place at even lower temperature attainable by recent cooling techniques.

Acknowledgments

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